-1 units overall

YEAR 12 MATHEMATICS APPLICATIONS(ATMAA) 2019
Test1: Growth and Decay in Sequences

SHENTON COLLEGE

NAME: SOLUTIONS

TOTAL
59

TEACHER: MACKENZIE MCRAE RYAN STAFFE

Calculator Free Section: No notes **Formula sheet provided** **Total time: 20 minutes**

29

QUESTION 1 [8 marks - 2, 2, 2, 2]

Write the first four terms of the following sequences: .

a). $T_{n+1} = T_n - 6, T_1 = -2$

$-2, -8, -14, -20$ ✓✓

b). $T_n = \frac{1}{4} T_{n-1}, \ T_1 = 8$

$8, 2, \frac{1}{2}, \frac{1}{8}$ ✓✓

c). $A_{n+1} = A_n - 8, A_3 = -8$

$8, 0, -8, -16$ ✓✓

d). $T_{n+2} = 2T_{n+1} + T_n, T_1 = 5$ and $T_2 = 8$.

$T_3 = 2(8)+5 = 21$

$T_4 = 2(21)+8 = 50$

$\therefore 5, 8, 21, 50$ ✓✓

Calculates correct terms, -1 any error.

QUESTION 2 [4 marks - 2, 2]

a) Last years attendance at the Travel Expo was 2000 more than the previous year, with the first years attendance being 20 000 people. Deduce a rule for the *nth* term of this arithmetic sequence.

$T_n = 20000 + (n-1)2000$ ✓ General Rule

OR $T_n = 2000n + 18000$ ✓ Correct substitution

b) John's ice-cream store sales were increasing by 1.6% per day due to rising temperatures over summer. Mondays ice-cream sales were $350, with Monday being the start of the week for John's business. The amount of ice-cream sales after the *nth* day can be modelled by a recurrence relation. Write the recurrence relation.

$T_{n+1} = 1.016 T_n, \quad T_1 = 350$

✓ Recurrence Rule

✓ Correct substitution

QUESTION 3 [8 marks – 4, 4]

a) Plot the graph of the sequence whose terms are -3, 5, 13, 21, 29... Identify whether the sequence is an arithmetic progression (AP) or a geometric progression (GP) and state the general rule.

AP or GP:

✓ Identifies rule type

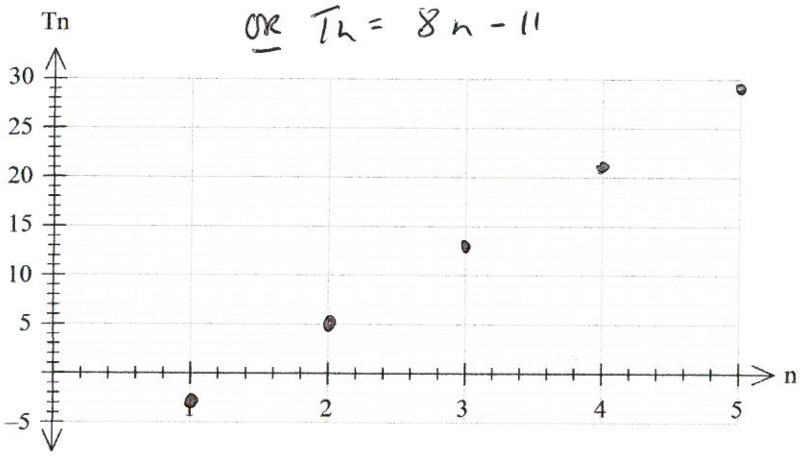
General Rule: $T_n = -3 + (n-1)8$

OR $T_n = 8n - 11$

✓ General Rule

✓ Correct Substitution.

✓ Plots accurately



b) Plot the graph of the sequence whose terms are 48, -24, 12, -6, 3.. Identify whether the sequence is an arithmetic progression (AP) or a geometric progression (GP) and state the general rule:

AP or GP:

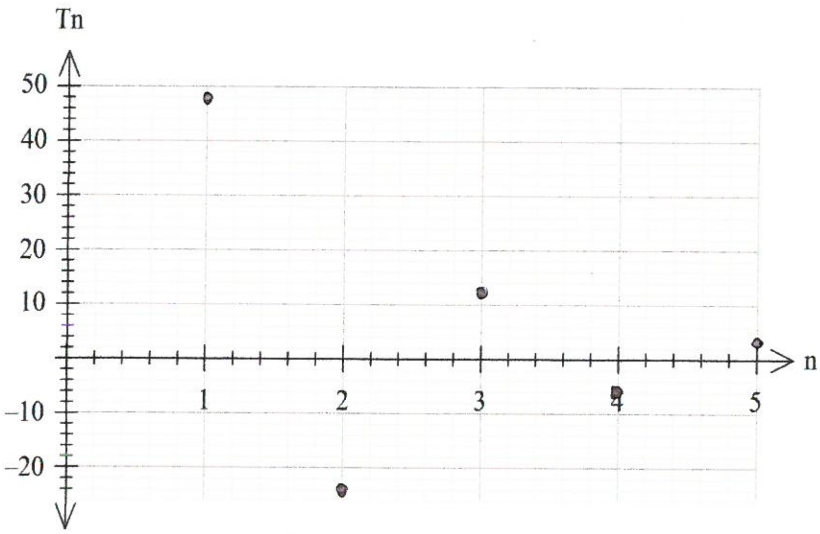
✓ Identifies Rule type

General Rule: $T_n = 48\left(-\frac{1}{2}\right)^{n-1}$

✓ General Rule

✓ Correct substitution

✓ Plots accurately



QUESTION 4 [4 marks - 1, 1, 2]

An arithmetic progression is such that $T_8 = 20$ and $T_{20} = 116$.

a) What is the common difference?

$$d = \frac{116-20}{12}$$

$$= \frac{96}{12} = 8$$

✓ Calculates difference.

b) State the first term of the sequence.

$$a + 7d = 20$$

$$a + 7(8) = 20$$

$$a = -36$$

✓ Calculates a

c) State the explicit rule of the sequence.

$$T_n = -36 + (n-1)8$$

OR $T_n = 8n - 44$

✓ Explicit Rule AP

✓ Correct substitution.

QUESTION 5 [5 marks -1,3, 1]

A new butterfly farm is being set up with 500 butterflies. The owners know that as visitors enter the farm some butterflies will fly away each year, however, they expect with breeding they will attain 30 new butterflies by the end of each year. They are confident this will increase their butterfly numbers over time. The amount of butterflies at the end of each year can be modelled by the following recurrence relation:

$$T_{n+1} = 0.9T_n + 30, \quad T_1 = 500$$

a) How would you describe the long term population of butterflies?

Butterfly population reaches a steady state.

✓ Describes long term solution.

b) Show how the butterfly farmers can calculate the expected butterfly numbers in the long run exactly and what will this be.

$$\frac{d}{1-r}$$

$$= \frac{30}{1-0.9}$$

$$= 300$$

$$x = 0.9x + 30$$

$$0.1x = 30$$

$$x = 300$$

$\therefore$ 300 butterflies

✓ Shows $t_{n+1} = t_n$ or $\frac{d}{1-r}$

✓ Use formula correctly

✓ Determines number of butterflies.

c) How many butterflies should the farmers hope to breed each year if they want butterfly numbers to remain at the 500 they set their farm up with?

$$0.1 \times 500$$

$= 50$ butterflies

✓ Calculates constant required.

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$\overline{}$
30

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Calculator Assumed **1 A4 page notes** **Formula sheet provided** **Total time: 30 minutes**

QUESTION 6 [7 marks – 2, 2, 2, 1]

Jared flies in a hot air balloon. He finds that he rises 155m during the first minute, 124m during the second minute, and each minute he is rising 80% of the height risen in the previous minute.

a) Complete the table below showing the distance Jarod's balloon rises during the third and fourth minute.

Number of flight minutes	1	2	3	4
Distance balloon rises (metres)	155	124	99.2	79.36

✓ ✓ Calculates 80% of previous.

b) Find the general rule for the distance the balloon is rising during the *nth* minute.

$$T_n = 155\ (0.8)^{n-1}$$

✓ General Rule

✓ Correct Substitution

c) Determine the distance Jared rises during the 8th minute to 2 decimal places.

$$T_8 = 155\ (0.8)^{(8-1)}$$

$$= 32.51 \text{ metres}$$

✓ Substitutes correctly for 8th minute.

✓ Distance to 2dp.

d) What is the maximum height(m) that Jared can reach in his balloon?

775 metres.

✓ Calculates maximum height

QUESTION 7 [7 marks - 2, 2, 1,2]

A frog is on one side of a dry creek that is 5.3m wide. It wants to reach the other side. The frogs first jump achieves a distance(D) of 82cm, however, each successive jump is 7cm less than the previous jump until it cannot jump forward before needing a rest stop.

a) List the sizes(cm) of the first three jumps.

82, 75, 68

✓✓ Continues progression

-1 any error.

b) Write the recursive rule for the frog's jumping distance, D_n in terms of D_{n-1}.

$D_n = D_{n-1} - 7 \quad D_1 = 82$

Recurrence relation w...

✓ Uses D_n in terms of D_{n-1}

✓ Correct substitution.

c) How far did the frog jump on the tenth jump?

$D_{10} = 19$cm

✓ Calculate D_{10}.

d) Verify whether the frog will reach the other side or not before it requires a rest stop.

$S_{12} = 5.22$m

$\therefore$ Frog will not reach other side without a rest stop.

✓ Provides evidence

✓ Correctly states with evidence.

QUESTION 8 [6 marks - 2, 2, 2]

Alice starts work as a doctor on a salary of \$86 000 in 2015. Each year thereafter, she expects to receive an increase in salary of 2.5% per year.

a) What will Alice's salary be during her fifth year of work ?

$T_n = 86000(1.025)^{n-1}$

$T_5 = \$94\,927.91$

✓ Identifies a and r.

✓ Determines salary

b) If Alice continues working in the same job in what year will she first receive a salary exceeding \$200 000?

$T_{36} = \$204\,095.65$

In 36th year OR 2050

✓ Determines term that exceeds \$200 000

✓ Determines year.

c) Another doctor, Lachlan, started work on a salary of \$80 000 and expects to receive an increase in salary of 3.2%p.a. In what year will his wage be greater than Alice's?

$T_n = 80000(1.032)^{n-1}$

$A_{12} = \$112\,839.45$

$L_{12} = \$113\,127.10$

$\therefore$ In the 12th year

OR 2026.

✓ Shows use of 2 rules.

✓ Determines year

QUESTION 9 [4 marks – 2, 1, 1]

Hayley visited the UWA research area where bacteria growth was being studied after n days. The number of bacteria present in a culture was increasing at a rate of 26% per day, however, with chemical control the bacteriologist was able to reduce bacteria numbers by 30 per day. An initial bacteria population of approximately 1200 was present.

a) Write a recurrence relation that gives the number of bacteria present after n days.

$T_{n+1} = 1.26\,T_n - 30\ ,\quad T_0 = 1200$

✓ Correct R.R. ✓ Identifies T_0

a) How many bacteria would be present after 1 week, to the nearest hundred?

$T_7 = 5583.90$

$\therefore \sim 5600$ bacteria ✓ Determines bacteria to nearest hundred.

b) It was feared that once the bacteria population reached 30 000 they would have lost control of its spread. How many days would it take for this to occur?

$T_{15} = 34855.71$ 15 days ✓ Determines no. of days.

QUESTION 10 [6 marks - 3, 2,1]

Tom thought his swimming pool may be leaking over summer as he noticed a crack on the bottom of his pool. He calculated he had 137 812.5 Litres of water in his pool after 3 weeks and 54 118.4 Litres of water after 10 weeks. He presumed his pool would continue to leak in this geometric pattern until he could have it repaired.

a) Show how Tom can find that the rate at which his pool is reducing in water per week is 12.5% to 3 decimal places.

$r^7 = \dfrac{54118.4}{137812.5}$

$r = 0.875$

$\therefore 12\tfrac{1}{2}\%$ loss.

✓ Identifies 7 week dif
✓ Shows ratio calc.
✓ Determines r is a 12.5% loss.

b) How much water would be left in Tom's pool after 13 weeks?

$a \times r^2 = 137812.5$

$a = 180000$

$180000 \times 0.875^{(13-1)}$

$T_{10} \times r^3 = 54118.4 \times 0.875^3$

$= 36255.10$ Litres

✓ Calculates a
✓ Calculates T_{13} in Litres

c) If Tom decides to wait until less than 1 Litre of water remains in his pool to fix the leak, how many weeks will he be waiting?

$T_1 = a = 180000$

$T_{92} = 0.9506$ Litres

$\therefore$ 92 weeks.

✓ Calculates number of weeks.